Heat Transfer

Unit II
Heat Conduction through Extended Surfaces(Fin)

Extended Surfaces (Fins)

- There are large no of engg equipment, where unutilized heat energy is required to be discarded to atm, otherwise systems will fail due to overheating

- Examples are cooling of IC engs, heat removal from nuclear reactors, ICs, electrical Tx, compressors, motors, refrigeration & air-conditioning systems, various types of heat exchangers, etc

- In most of the above cases, heat is rejected from solid surfaces by convection to atm (low h values)

- Our aim would be to increase heat transfer rate so that temp of solid surface is maintained within desired limits to avoid failure of the system

- We know that heat flow by convection :

$$Q = hA(T - T_{\infty});$$

  - where h is almost constant (5 to 12 W/m^2K), whenever heat is convected to atm & can not be controlled
  - $(T - T_{\infty})$ is temp diff between solid surface & atm, which also can not be controlled

- So, the only way to increase heat transfer rate Q is by increasing the surface area A

- Surface area of solid is increased by providing extended surfaces called FINS

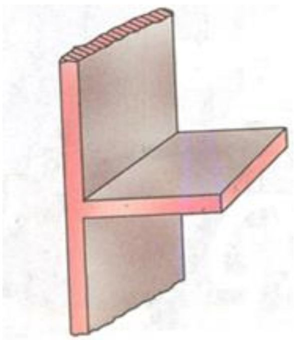
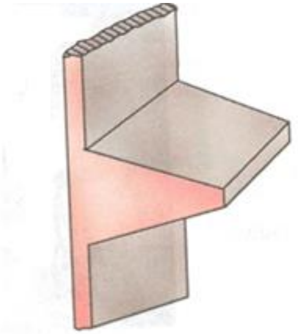
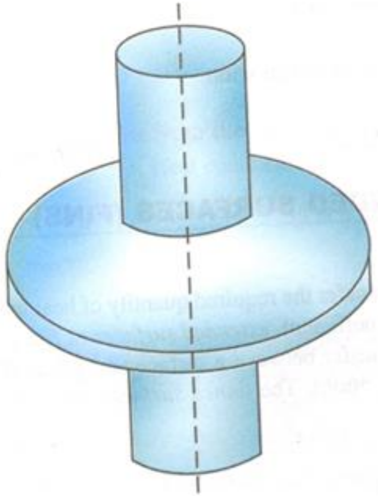


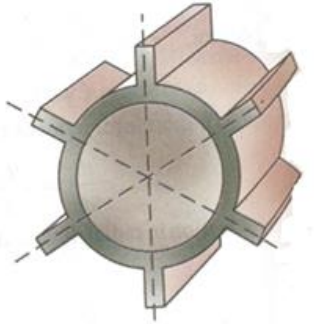
Plate Fin



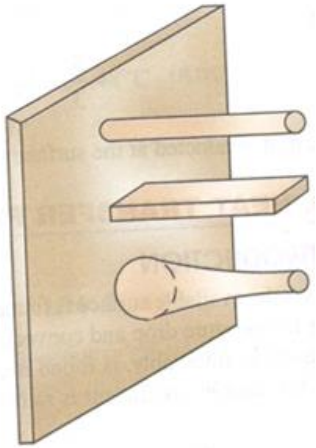
Tapered Fin



Disk Fin



Radial Plate Fins



Pin Fins

Different Types Of Fins

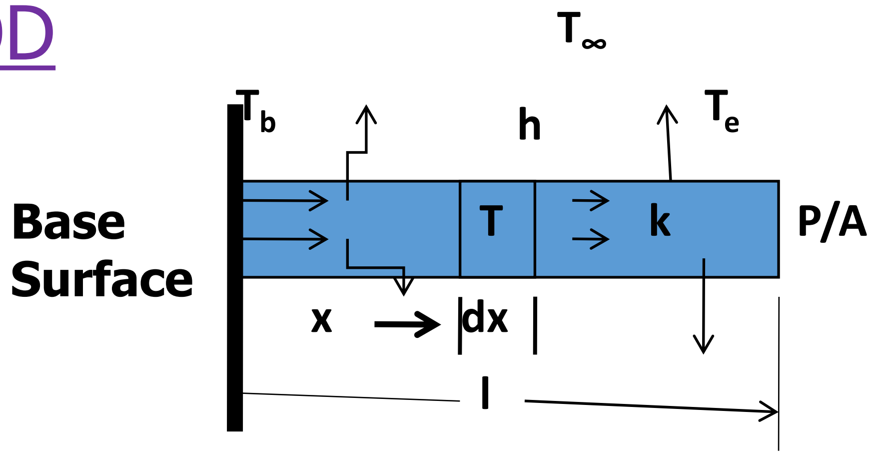
Analysis of PIN FIN/THIN ROD

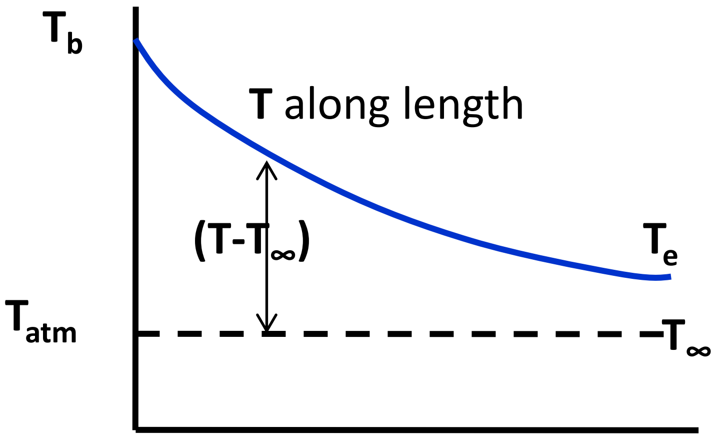
- Consider a pin fin of uniform Cross sectional area of A (perimeter P)

- Fin is connected to base surface, which is at temp T_b and from where heat is to be removed

- Heat transfer from base surface to fin and through the fin is by conduction and to atm/surrounding fluid from fin surface by convection

- Temp variation along the length of fin is shown in Fig; It reduces along the length away from base



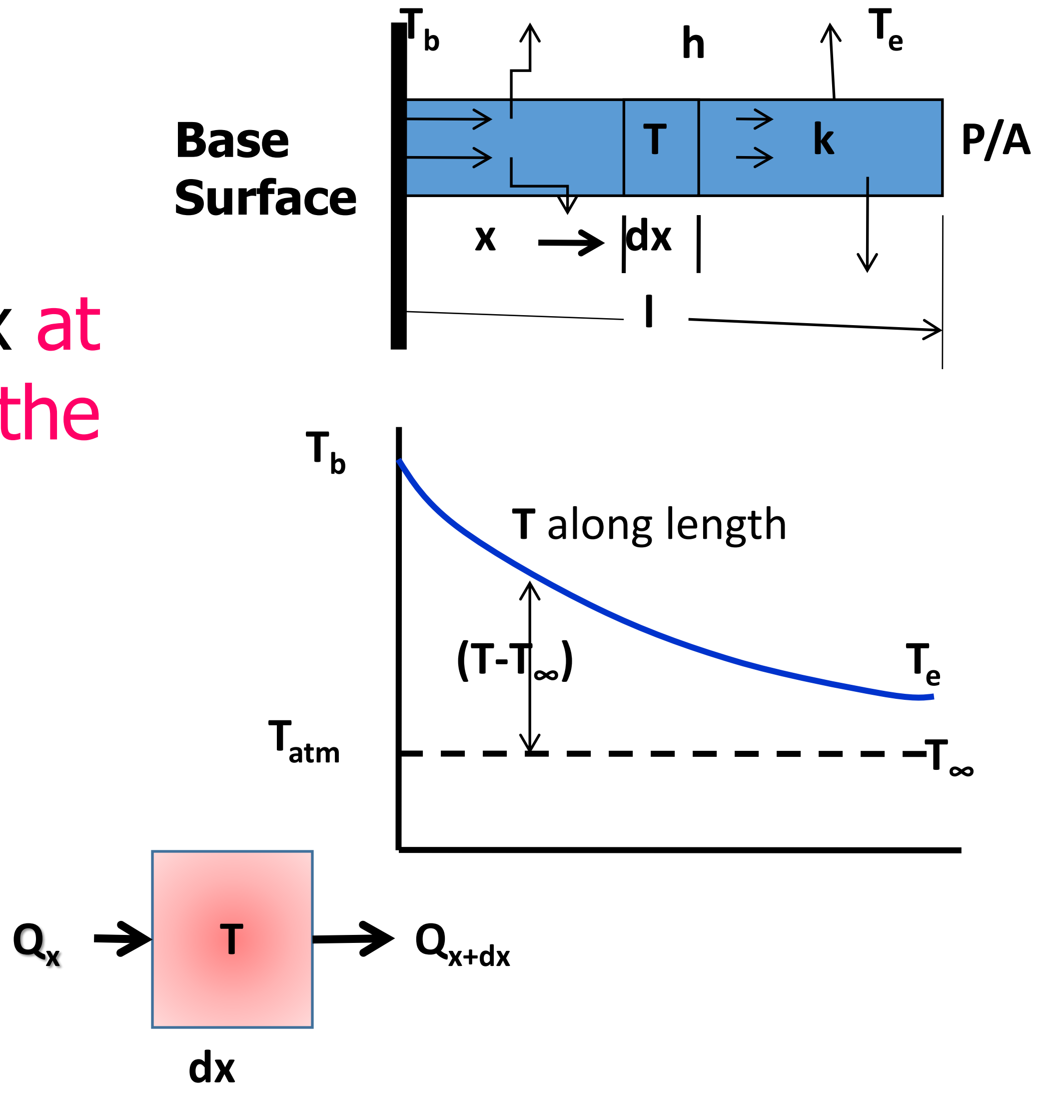


- Consider heat flow to and from an elemental section of length dx at a distance x from the base at temp T

- Let heat entering elemental section dx at face x of area A be Q_x which shall be the heat conducted into the element

- Hence Q_x can be given as:

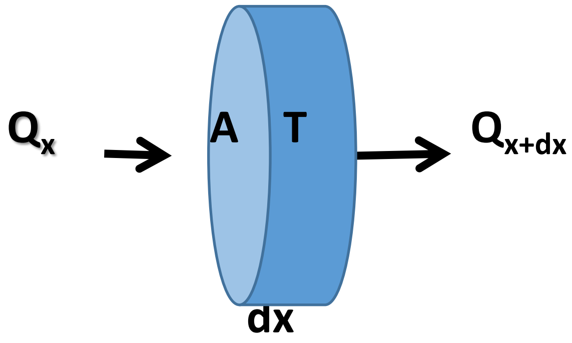
$$Q_x = -kA\frac{dT}{dx} \quad from\ Fourier's\ Law$$



- And heat conducted out from element at face x+dx will be Q_{x+dx}

- We know that:

$$\frac{d}{dx}(Q_x) = \frac{Q_{x+dx} - Q_x}{dx};$$

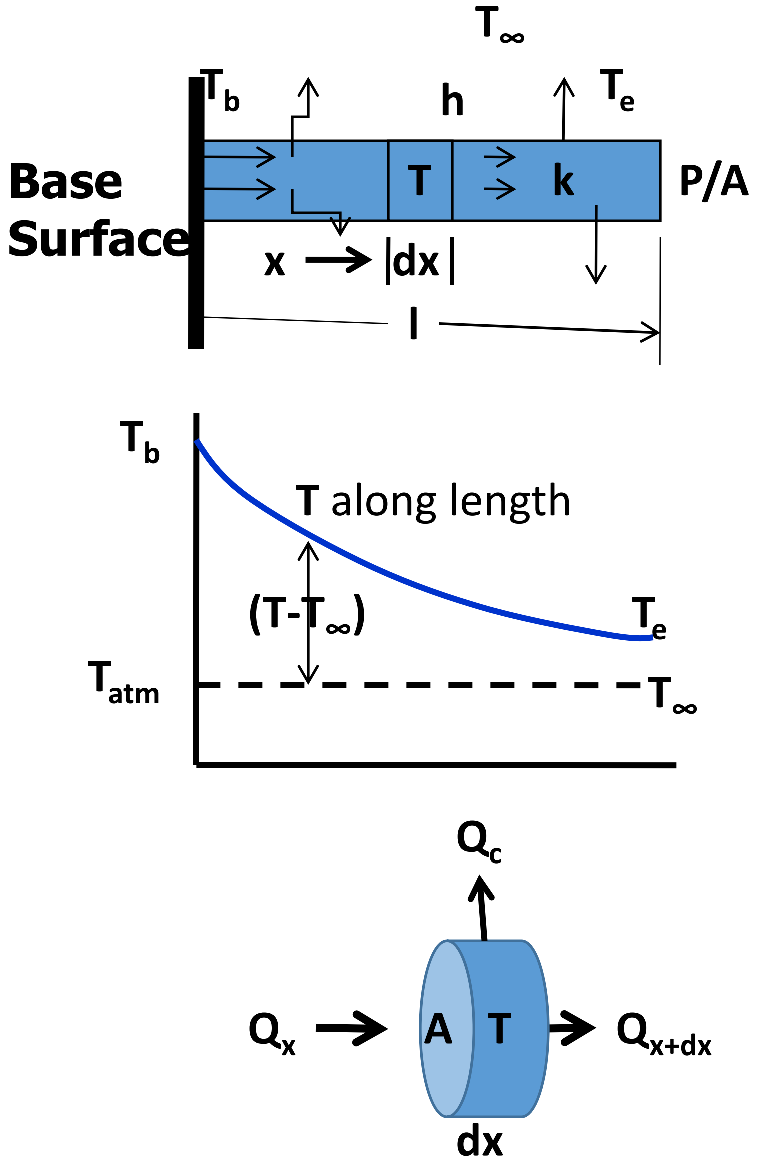


$$hence \quad Q_{x+dx} = Q_x + \frac{d}{dx}(Q_x)dx$$

Analysis of PIN FIN/THIN ROD

- And heat convected out from surface of element dx to surroundings will be:

  $Q_c = h.P.dx\ (T - T_\infty)$

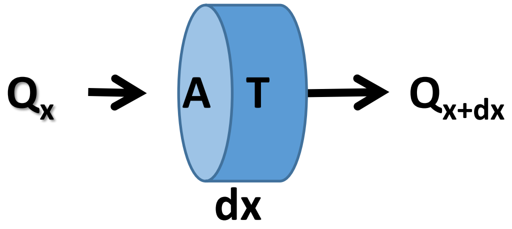


• Writing energy balance eqn. for element dx :

Heat conducted in to the element = Heat conducted out + Heat convected out of element

$$Q_x = Q_x + \frac{d}{dx}(Q_x)dx + hPdx(T - T_\infty) \quad OR$$

$$0 = \frac{d}{dx}(Q_x)dx + hPdx(T - T_\infty) \quad OR$$



$$0 = \frac{d}{dx}(-kA\frac{dT}{dx})dx + hPdx(T - T_\infty) \quad OR$$

$$kA\frac{d^2T}{dx^2}dx = hPdx(T - T_\infty) \quad OR$$

$$\frac{d^2T}{dx^2} = \frac{hP}{kA}(T - T_\infty)$$

- Putting $(T - T_\infty) = \theta$; the excess temp and let $hP/kA = m^2$; We have :

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

We have :

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

- This is the second order differential equation for temp distribution along the fin, whose general solution is of the form:

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

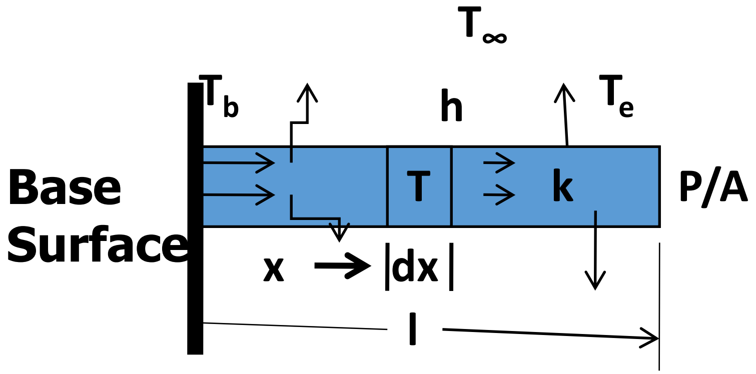
$$and\ the Slope\ \frac{d\theta}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

Case – I : Adequately Long FIN (Fin with Insulated End/Tip)

Boundary Conditions:

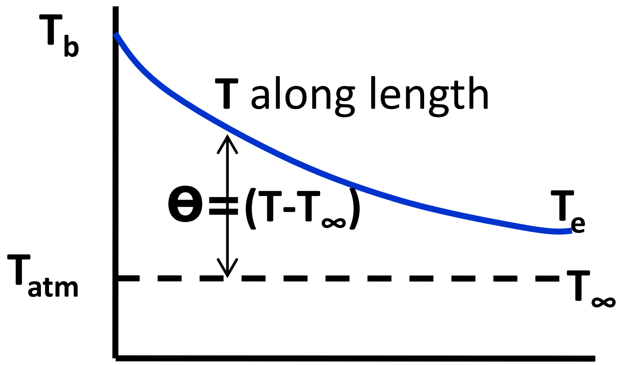
1. At x=0; $T=T_b$;$\theta=\theta_b$ 2. At x=l; Q=0; $d\theta/dx=0$

(As no heat flow by convection from Tip, so assumed insulated Actually, there is no insulation)



- Applying BC 1) we get:

$$C_1 + C_2 = \theta_b \ldots\ldots\ldots\ldots\ldots(1)$$



Applying BC 2), We have:

$$\frac{d\theta}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

$$\left[\frac{d\theta}{dx}\right]_{x=l} = mC_1 e^{ml} - mC_2 e^{-ml} = 0$$

$$\Rightarrow C_1 e^{ml} - C_2 e^{-ml} = 0 \ldots\ldots\ldots\ldots(2)$$

From Equations (1) & (2); we have:

$$C_1 = \frac{\theta_b e^{-ml}}{e^{ml} + e^{-ml}} \text{ and } C_2 = \frac{\theta_b e^{ml}}{e^{ml} + e^{-ml}}$$

$$Substituting\ C_1\ \&\ C_2\ in\ Eqn\ \theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$\frac{\theta}{\theta_b} = \frac{e^{-ml}.e^{mx} + e^{ml}.e^{-mx}}{e^{ml} + e^{-ml}}$$

$$\frac{\theta}{\theta_b} = \frac{e^{m(l-x)} + e^{-m(l-x)}}{e^{ml} + e^{-ml}} = \frac{Cosh\ \ m(l-x)}{Cosh\ \ ml}$$

$$Also,\ \theta\ at\ the\ tip\ will\ be\ \frac{\theta_e}{\theta_b} = \frac{1}{Cosh\ ml}$$

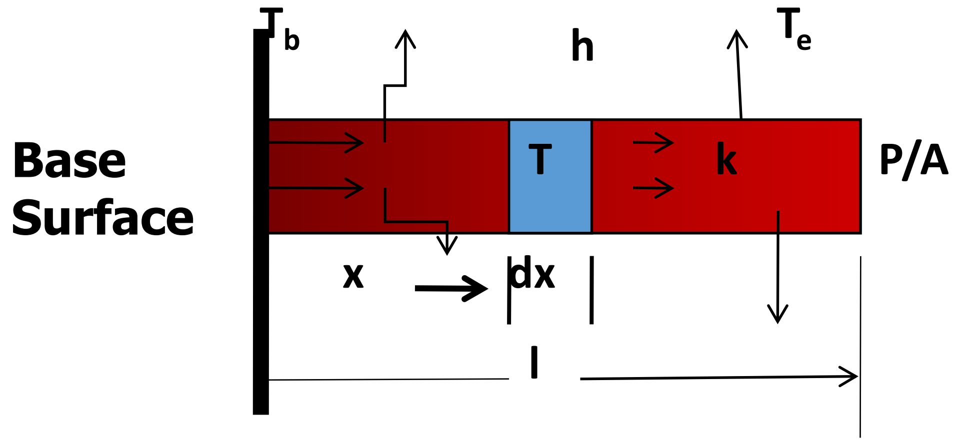
Heat Flow Rate from Fin: Heat conducted to Fin at the base shall be the heat Flow rate from Fin

$$Hence\; Q = -kA\frac{d\theta}{dx}\; at\;\; x = 0$$

$$Substituting\;\; \frac{d\theta}{dx}\;\; at\;\; x = 0;\; We\; have$$

$$Q = kAm\,\theta_b\,\tanh ml$$

$$OR\;\;\; Q = \theta_b\sqrt{hPkA}\;\tanh ml$$



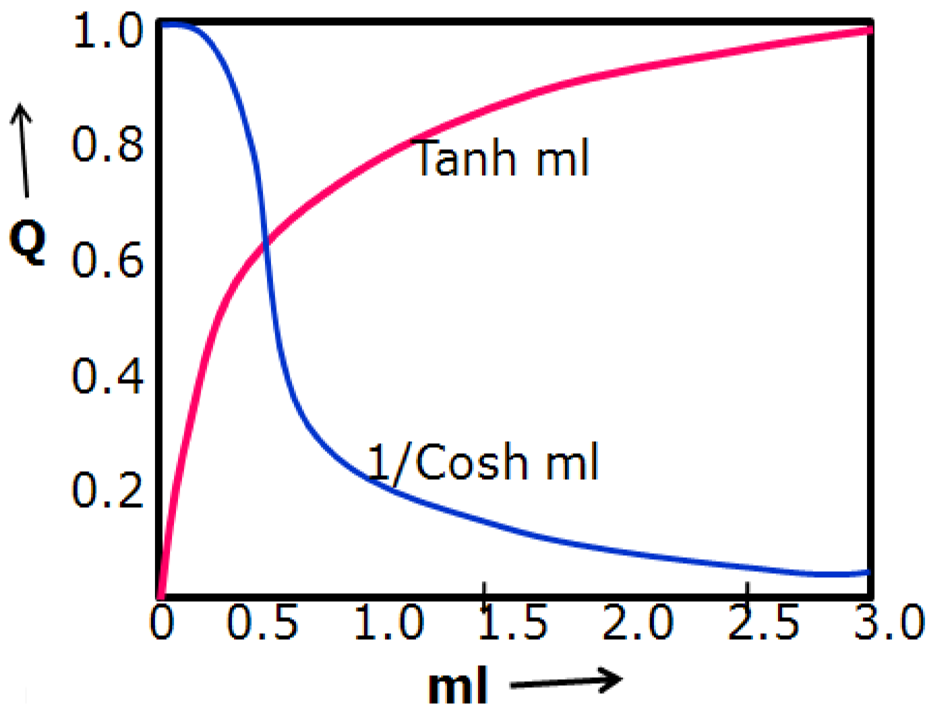
Case – I : Adequately Long FIN (Fin with Insulated End/Tip)

What is adequately long Fin?

Let us look at Q v/s ml & θ v/s ml plots

As ml or l increases, tanh ml first increases rapidly and then becomes asymptotic at ml ≈3.

Also, θ_e approaches zero at ml ≈3.



Thus, increasing ml (or l) beyond 3 will not give any advantage. Hence Fin with ml≈3 is called adequately long fin or fin with insulated tip/end

Case –II : Analysis of Very Long FIN

Boundary Conditions:

1. At x=0; $T=T_b$; $\theta=\theta_b$
2. At x=l; $\theta=\theta_e=0$

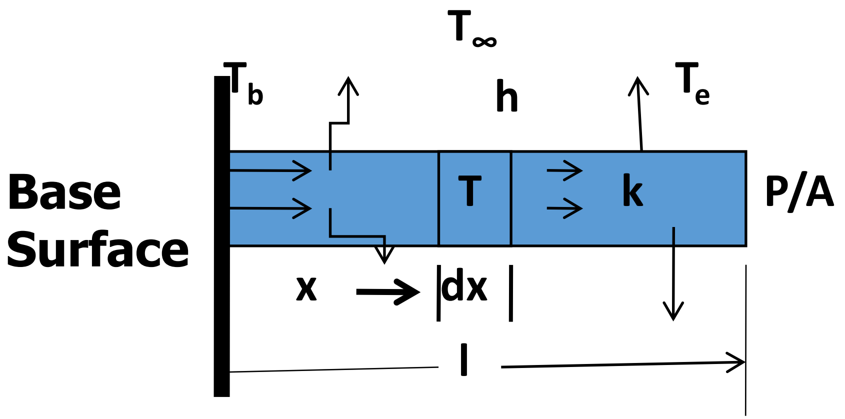
- Applying above BC 1), we get:

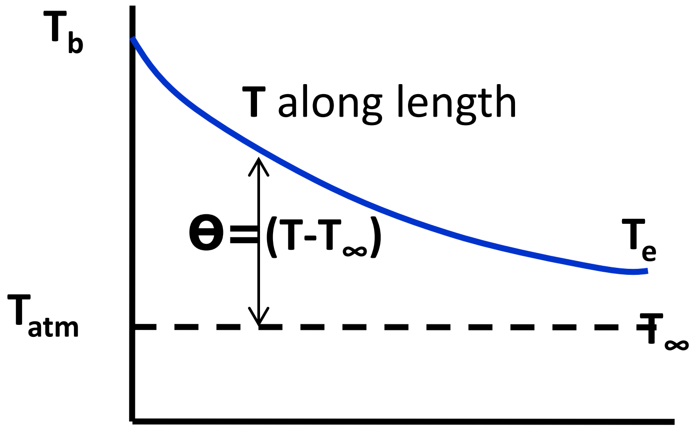
$$\theta_b = C_1 + C_2 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Applying BC 2); We have;

$$\theta = C_1 e^{ml} + C_2 e^{-ml} = 0\ldots\ldots(2)$$

$$C_1 = \frac{-\theta_b e^{-ml}}{e^{ml} - e^{-ml}} \quad \& \quad C_2 = \frac{\theta_b e^{ml}}{e^{ml} - e^{-ml}}$$





Case –II : Analysis of Very Long FIN

$$Substituting\ C_1\ and\ C_2\ in\ Eqn\ \theta = C_1 e^{mx} + C_2 e^{-mx};$$

$$\frac{\theta}{\theta_b} = \frac{-e^{-ml}.e^{mx} + e^{ml}.e^{-mx}}{e^{ml} - e^{-ml}} = \frac{e^{m(l-x)} - e^{-m(l-x)}}{e^{ml} - e^{-ml}}$$

$$\frac{\theta}{\theta_b} = \frac{Sinh\, m(l-x)}{Sinh\, ml}$$

$$Also,\ \theta = C_1 e^{ml} + C_2 e^{-ml} = 0......(2)$$

$$Substituting\ l = \infty : C_1 e^{m\infty} + C_2 e^{-m\infty} = 0$$

$$or\ C_1 e^{\infty} + 0 = 0$$

$$\Rightarrow C_1 = 0$$

$$Hence\ \ C_2 = \theta_b\ \ from\ eqn...(1)\ C_1 + C_2 = \theta_b$$

$$Substituting\ C_1\ and\ C_2\ in\ Eqn\ \theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$Therefore,\ we\ also\ have\quad \theta = \theta_b e^{-mx}$$

Heat Flow Rate through Fin:

$$Q = -kA\left[\frac{d\theta}{dx}\right]_{x=0} = -kA(-m)\theta_b \frac{Cosh\ \ ml}{Sinh\ \ ml}$$

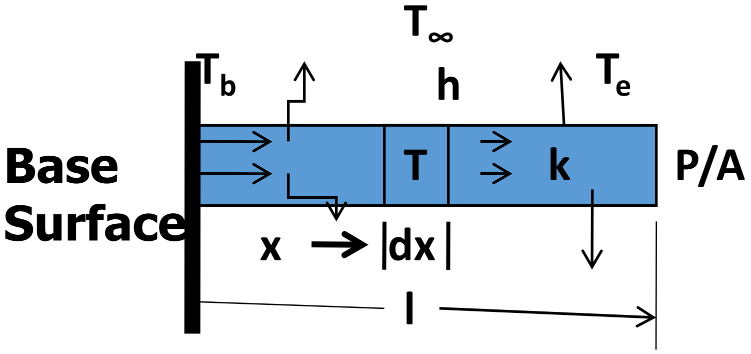
$$Q = kAm\theta_b Coth\ ml \approx kAm\theta_b$$

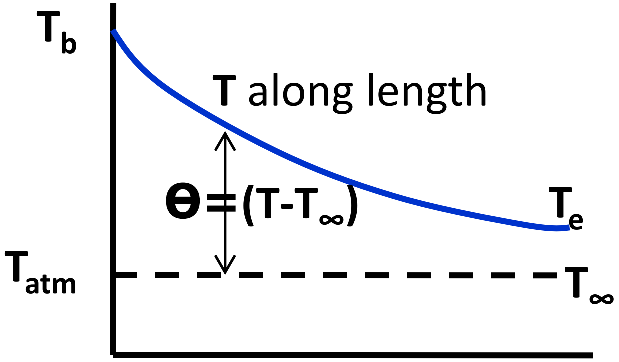
$$Also,\ Q = \theta_b \sqrt{hPkA}\, Coth\ ml$$

Case – II : Analysis of Short FIN

Boundary Conditions:

1. At x=0; $T=T_b$;$\theta=\theta_b$
2. At x=l; Heat conducted to Fin Tip= Heat convected from Fin Tip





$$That\ \ is : -kA\left[\frac{d\theta}{dx}\right]_{x=l} = \left[hA\theta\right]_{x=l}$$

- Applying above BC 1), we get:

$$\theta_b = C_1 + C_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Case – II : Analysis of Short FIN

Applying BC2) in:

$$-kA\left[\frac{d\theta}{dx}\right]_{x=l} = [hA\theta]_{x=l}$$

$$-kA\left[mC_1e^{ml} - mC_2e^{-ml}\right] = hA\left[C_1e^{ml} + C_2e^{-ml}\right] .(2)$$

$$From\ \ Eqn\ (1)\ \&\ (2);$$

$$C_1 = -\left[\frac{\frac{h}{mk}-1}{\frac{h}{mk}+1}\right]\frac{\theta_b e^{-ml}}{e^{ml} - \left[\frac{\frac{h}{mk}-1}{\frac{h}{mk}+1}\right]e^{-ml}} \quad \& \quad C_2 = \frac{\theta_b e^{ml}}{e^{ml} - \left[\frac{\frac{h}{mk}-1}{\frac{h}{mk}+1}\right]e^{-ml}}$$

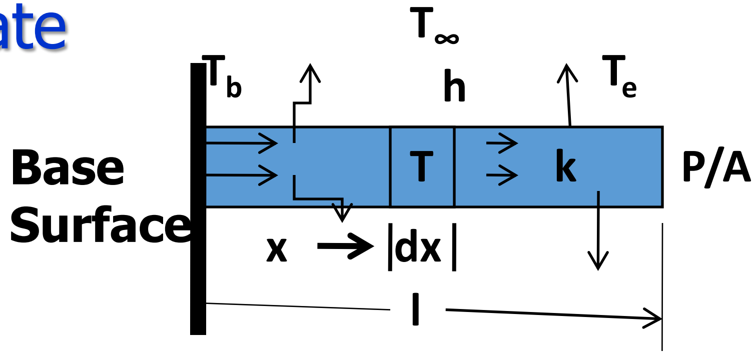
Putting C_1 & C_2 in eqn: $\theta = C_1 e^{mx} + C_2 e^{-mx}$

$$We\ have\quad \frac{\theta}{\theta_b} = \frac{Cosh\, m(l-x) + \frac{h}{mk} Sinh\, m(l-x)}{Cosh\, ml + \frac{h}{mk} Sinh\, ml}$$

$$And\quad \frac{\theta_e}{\theta_b} = \frac{1}{Cosh\, ml + \frac{h}{mk} Sinh\, ml}$$

Heat Flow Rate from Fin:

Heat conducted to Fin at the base shall be the heat Flow rate from Fin



$$Hence\ Q = -kA\frac{d\theta}{dx}\ at\ x = 0 \quad Subsituting\ \frac{d\theta}{dx}\ at\ x = 0$$

$$Q = kAm\theta_b\left(\frac{\frac{h}{mk} + \tanh ml}{1 + \frac{h}{mk}\tanh ml}\right) = \theta_b\sqrt{hPkA}\left(\frac{\frac{h}{mk} + \tanh ml}{1 + \frac{h}{mk}\tanh ml}\right)$$

Rectangular Fins

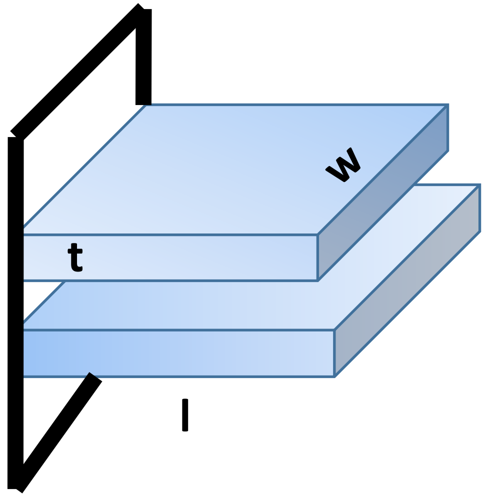
P=2t+2w;

For t<<w;

P=2w

And A=t x w

$$Hence\ m=\sqrt{\frac{Ph}{kA}}=\sqrt{\frac{2wh}{ktw}}=\sqrt{\frac{2h}{kt}}$$

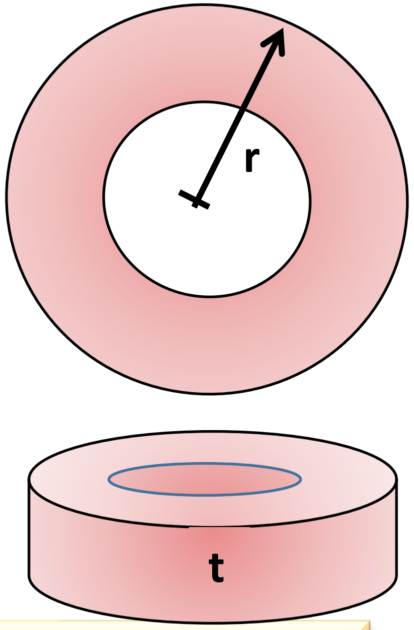


Circular/Disc Fins

P=2πr x 2;

And A=2πr x t

$$Hence\ m=\sqrt{\frac{Ph}{kA}}=\sqrt{\frac{4\pi rh}{k2\pi rt}}=\sqrt{\frac{2h}{kt}}$$



Fin Effectiveness

Fin Effectiveness (E) is defined as the ratio of Actual Heat Transfer Rate from finned surface to the Heat Transfer Rate from the area blocked by fin (or When fin was not there)

Naturally, use of fins is justified only when Effectiveness is greater than one (and when h is very low)

$$E = \frac{Q_{withFin}}{Q_{w/oFin}} = \frac{\theta_b \sqrt{PhkA}\tanh ml}{hA\theta_b} = \frac{\tanh ml}{\sqrt{\frac{hA}{Pk}}}$$

Fin Effectiveness (E)

$$E = \frac{Q_{with\,Fin}}{Q_{w/o\,Fin}} = \frac{\theta_b \sqrt{PhkA}\tanh ml}{hA\theta_b} = \frac{\tanh ml}{\sqrt{\frac{hA}{Pk}}}$$

When ml is large (≥3), tanhml tends to become 1.

$$Hence \quad E = \sqrt{\frac{kP}{hA}};$$

It should be greater than 1 for fin to be effective

$$E = \frac{Q_{withfin}}{Q_{w/o\,Fin}} = \frac{\eta_{fin} Q_{max}}{Q_{w/o\,Fin}} = \frac{\eta_{fin} A_f h\theta_b}{hA\theta_b} = \frac{\eta_{fin} A_f}{A}$$

Fin Efficiency (η)

Fin Efficiency (η) is defined as the ratio of Actual Heat Transfer Rate to max possible heat transfer rate from the same fin.

Heat Transfer Rate Q from fin shall be max when fin material has infinite conductivity k so that temp of fin all along the length can be assumed to be same as that at the base of fin

So $Q_{max} = h. A_f .\Theta_b = h.P.l. \Theta_b$

$Q_{actual} = \Theta_b \sqrt{hPkA} \tanh ml$ for sufficiently long fin

Fin Efficiency (η)

$$Hence \quad \eta = \frac{\theta_b \sqrt{hPkA}\tanh ml}{hPl\theta_b} = \frac{\tanh ml}{\sqrt{\frac{Ph}{kA}}.l} = \frac{\tanh ml}{ml}$$

for fin with insulated end

$$Fin\,Efficiency\,for\,Short\,Fin$$

$$\eta = \frac{\frac{h}{mk} + \tanh ml}{ml\left(1 + \frac{h}{mk}\tanh ml\right)}$$

$$Fin\,Efficiency\,for\,Long\,Fin\;\eta = \frac{1}{ml}$$

Overall Fin Effectiveness

Overall Fin Effectiveness for a finned surface is defined as:

$$E_{overall} = \frac{Total\ Heat\ Transfer\ from\ Finned\ Surface}{Heat\ Transfer\ from\ the\ base\ if\ there\ were\ no\ fins}$$

$$E_{overall} = \frac{h\left(A_{unfin} + \eta_{fin} A_{fin}\right)\left(T_b - T_\infty\right)}{h.A_{no\ fin}\left(T_b - T_\infty\right)}$$

Overall Fin Effectiveness thus depends on the fin density and individual fin effectiveness

Overall Fin Effectiveness is better measure of performance of a finned surface than effectiveness of individual fins

Error in Temp Measurement by Thermometer

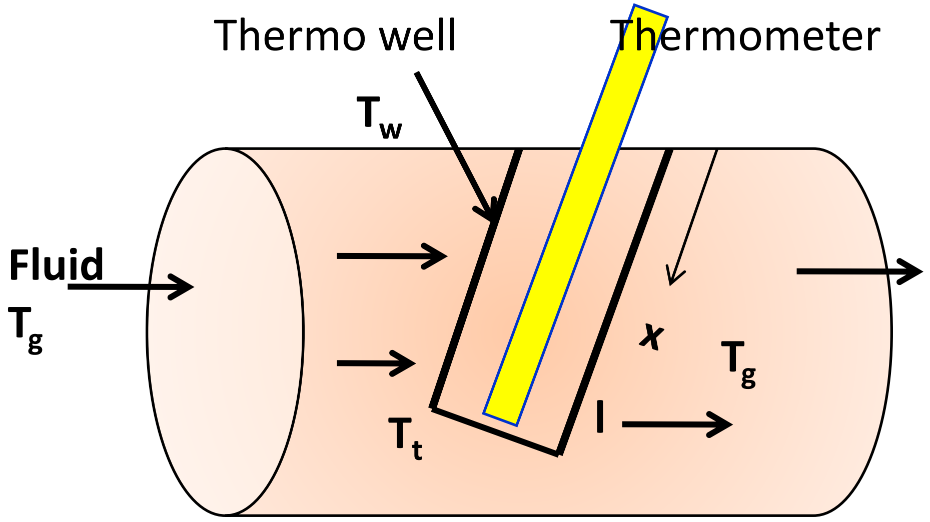
Principle: From temp distr. for fin, we know that temp along the length of fin approaches surrounding fluid temp & at the end of Fin, it shall be nearest to fluid temp.

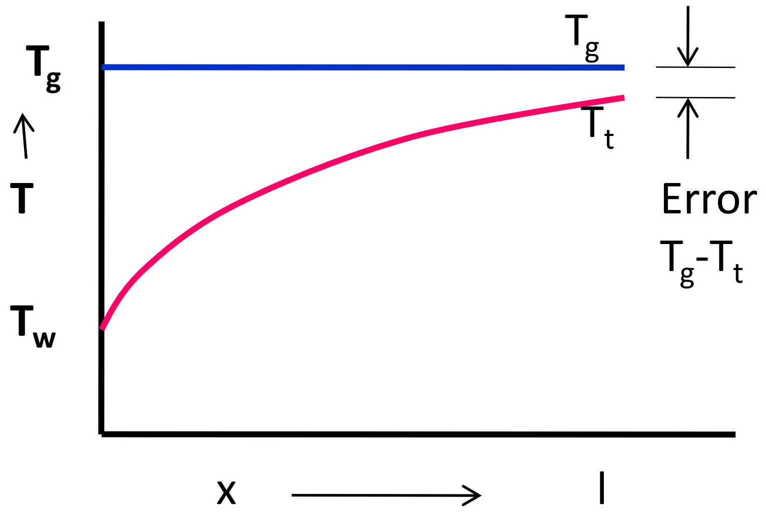
We also know that temp at the end of fin will become same as that of surrounding fluid, only when length of fin will be infinite.

Since infinite length of fin is not possible practically, the recorded temp T_t is always different (lower) from actual fluid temp T_g

This is known as Error in Temp Measurement

As shown in Graph, Error in temp measurement shall be (T_g - T_t). T_w is the wall Temp of the conduit





Error in Temp Measurement by Thermometer

Thermo well can be assumed as FIN and hence applying fin analysis, actual temp of fluid can be found by estimating the error.

Expression for temp distribution for adequately Long Fin

$$\frac{\theta}{\theta_b} = \frac{Cosh\,m(l-x)}{Cosh\,ml} \quad : where\ l\ is\ the\ Length\ of\ Thermowell$$

Temp at the End of Fin, that is Thermowell,

$$\theta_{end} = T_g - T_t \quad for\ x = l\ can\ be\ written\ as$$

$$\frac{T_g - T_t}{T_g - T_w} = \frac{1}{Cosh\,ml}$$

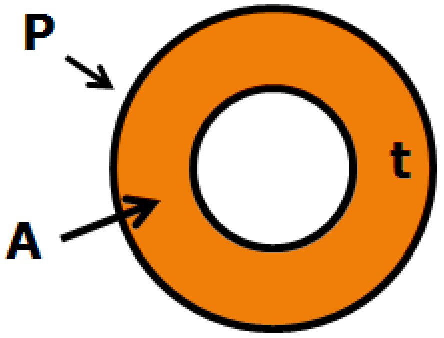
Here $(T_g - T_t)$ *is the Error in Temp Measurement*

Error in Temp Measurement by Thermometer

$$\frac{T_g - T_t}{T_g - T_w} = \frac{1}{Coshml}$$

$$And \quad T_g - T_t \propto \frac{1}{Coshml} \propto \frac{1}{ml}$$

$$Because,\ with\ increase\ in\ ml,\ Coshml\ increases$$



If D is outer dia of Thermowell and t the thickness of its wall and assuming D>>t,

Then Perimeter P = лD & Area A = лDt

Error in Temp Measurement by Thermometer

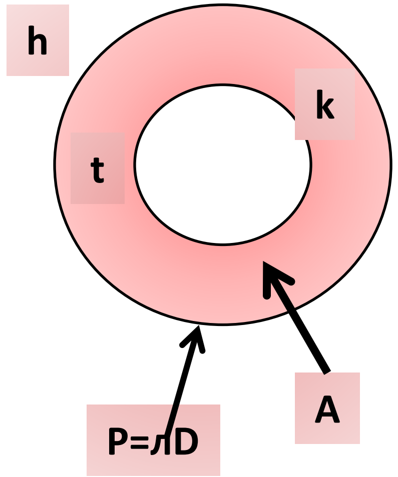
If D is outer dia. of Thermowell and t the thickness of its wall and assuming D>>t,

$$m = \sqrt{\frac{Ph}{kA}} = \sqrt{\frac{h.\pi D}{k.\pi Dt}} = \sqrt{\frac{h}{kt}}$$

$$Where\,P = \pi D\, as\, only\, outside\, surface$$

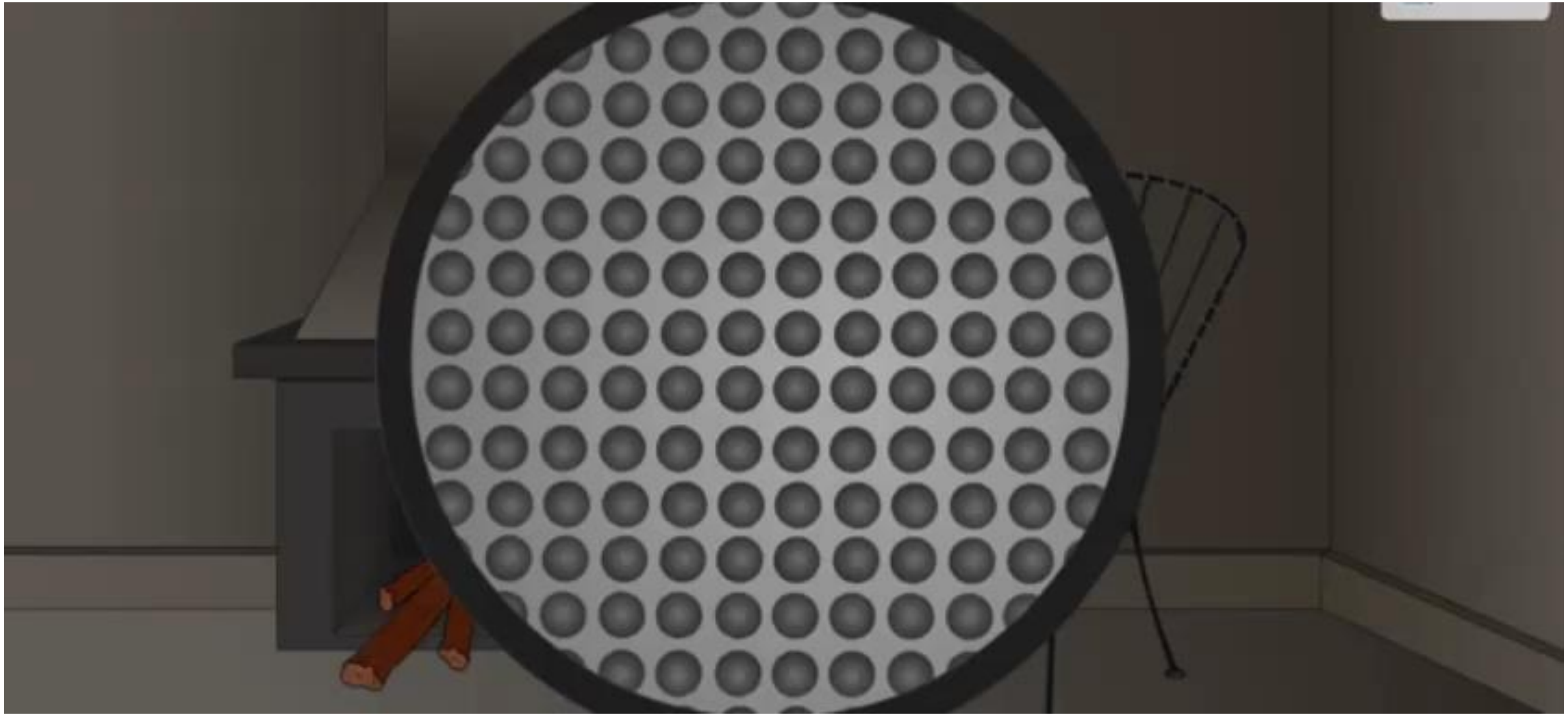
$$of\,\, well\, is\, receiving\, heat\, and\, not\, inside$$

$$Then\, Error\, T_g - T_t \propto \frac{1}{ml} \propto \frac{1}{l\sqrt{\frac{h}{kt}}}$$



So, to minimize error in measurement (T_g-T_t);

1. Length of fin (l) should be as long as possible
2. Thickness of thermowell wall (t) should be as small as possible

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